Confidence Intervals

Terminology

Confidence Interval (CI) — A range of values that have a stated probability (the *degree of confidence*) of containing the actual value of a population parameter.

Point Estimator — A statistic used as the value for a population parameter; the value of the statistic is the *point estimate*.

Critical Value (z^*, z_{crit}) — The *Z*-value that represents a stated margin of error.

Z^* (Z_{crit}) Values

Confidence	*Z**
90%	1.645
95%	1.96
98%	2.33
99%	2.58

Calculating a CI

The CI calculation is calculated from a point estimate and a margin of error:

$$\text{CI} = \text{point estimate} \pm \text{margin of error}$$

Validity requirements

These equations are valid if:

Random data

10% rule

$n \le 0.1N$

Large counts

$n \cdot \hat{p} \ge 10$

$n(1-\hat{p}) \ge 10$

CI for Population Proportions

- Confidence Interval for a proportion *p* at a given confidence level is:

$$\text{CI} = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

CI for Population Means

When σ is known

$$\text{CI} = \bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$\bar{x}$ - *Sample mean;* z^* - *critical z-value;* σ - *population std. dev.;* n - *sample size*

When σ is not known

$$\text{CI} = \bar{x} \pm t^* \frac{s_x}{\sqrt{n}}$$

$\bar{x}$ - *Sample mean;* t^* - *critical t-value;* s_x - *sample std. dev.;* n - *sample size*

Calculator Note

On your graphing calculator, three functions are associated with confidence intervals:

Proportions

- *1-PropZInt* z-interval

Means

- *TInterval* t-interval
- *ZInterval* z-interval

General

- *InvT, InvNorm* find *t**, *z**

Margin of Error and Standard Error

- Margin of error - half the size of the CI; that is, the "±" part of the CI equations.
- Standard error - The margin of error without the *t** or *z** element.

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Choosing Sample Size

To choose the sample size needed to achieve a particular margin of error, solve the margin of error equation for *n*.

Population proportions	*Population Mean (σ known)*	*Population Mean (σ unknown)*
$n = \hat{p}(1-\hat{p})\left(\frac{z^*}{ME}\right)^2$	$n = \left(\frac{z^*\sigma}{ME}\right)^2$	$n = \left(\frac{t^*s_x}{ME}\right)^2$

- If you are missing $\hat{p}$ for population proportion intervals, assume $\hat{p} = 0.5$; this is a conservative estimate.
- Round the resulting number up to the next higher integer.

Interpreting Confidence Interval and Level

The following are the interpretation of the confidence interval and the confidence level of the following study:

- A random sample of 3,000 U.S. adults were asked if they had ever seen a live dinosaur; of the sample, 16% responded "yes." The 95% confidence interval for this was determined to be (.143, .197).

The interpretations of this are as follows:

Confidence Interval

Generic "We are ___% confident that the interval from ____ to ____ captures the true proportion of _____."

Example We are 95% confident that the interval from .143 to .197 captures the true proportion of adults who have seen a live dinosaur.

Confidence Level

Generic "If many samples of _*sample size*_ are taken, the resulting confidence intervals would capture the true proportion of ___test question___ for about ____% of those samples."

Example If many samples of 3,000 US adults were taken, the resulting confidence intervals would capture the true proportion of adults who have seen a live dinosaur for about 95% of those samples.